Block Diagrams and Stability

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Agenda

1. Introduction
2. Block Diagrams
3. Control Loop Stability
   - ✓ Effect of Gains
   - ✓ Effect of Time Constants
   - ✓ Effect of Dead Time
   - ✓ Effect of Integral Action in the Controller
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Introduction

This chapter presents a discussion of block diagrams and control loop stability. It is important to present the development of block diagrams because they are used in the study of stability, in the design of feedforward controllers, in understanding the Smith predictor dead-time compensation, and in understanding multivariable control The presentation of stability is done minimizing the mathematics and emphasizing the physical significance.

Block Diagrams

Block diagrams show graphically how the process units and the instrumentation interact to provide closed-loop control. These diagrams are composed of three elements.

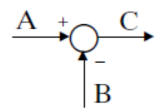
1. Arrows: The arrows indicate either variables or signals.
2. Blocks: Every block has an input, I, and an output, O. Inside the box we write the equation that describes how the input affects the output. In control work, this equation is a transfer function. Remember: The transfer function tells us how the input affects the output; that is,

transfer function → how much the input affects the output (K)

transfer function → how fast the input affects the output (τ, t_o)

Block Diagrams

3. Circles: Circles have at least two inputs. They represent the algebraic summation of the inputs, or C = A - B.



Let us now look at the development of block diagrams for two processes.

Block Diagrams

Example 6-1.1. Consider the heat exchanger control system shown in Fig. 6-1.1.

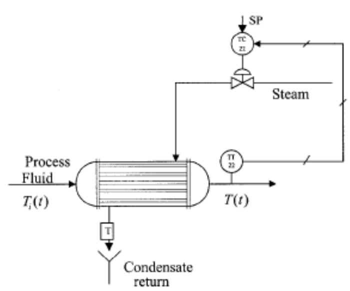


Figure 6-1.1 Heat exchanger control system.

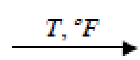


Figure 6-1.2 Arrow representing the controlled variable in engineering units.

Block Diagrams

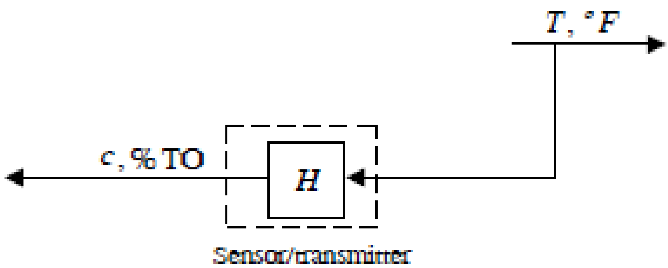


Figure 6-1.3 Block diagram showing sensor/transmitter.

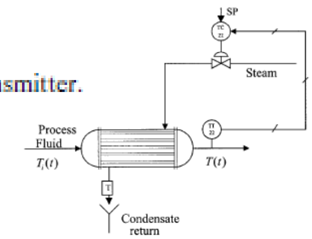


Figure 6-1.1 Heat exchanger control system.

Block Diagrams

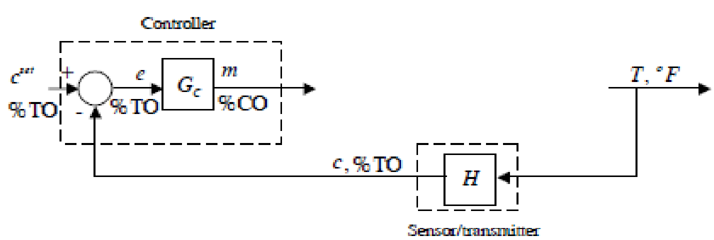


Figure 6-1.4 Block diagram with controller added.

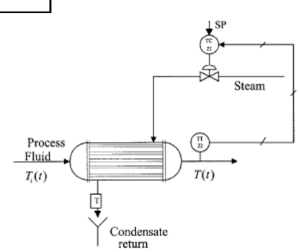


Figure 6-1.1 Heat exchanger control system.

Block Diagrams

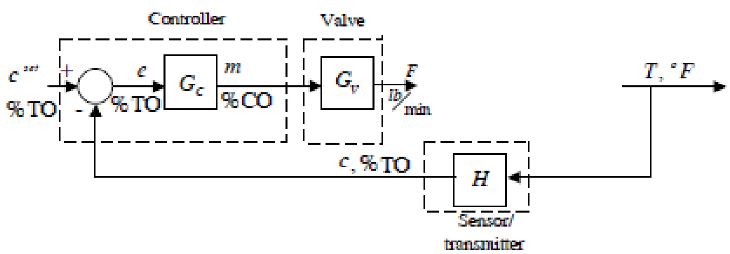


Figure 6-1.5 Block diagram with valve added.

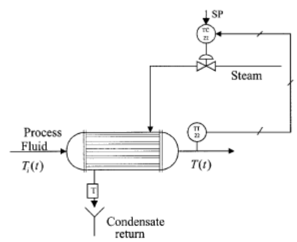


Figure 6-1.1 Heat exchanger control system.

Block Diagrams

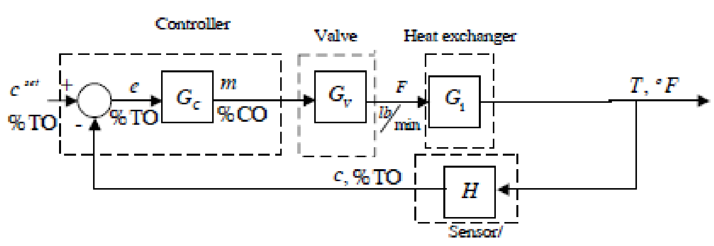


Figure 6-1.6 Block diagram showing control loop.

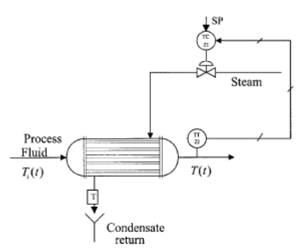


Figure 6-1.1 Heat exchanger control system.

Block Diagrams

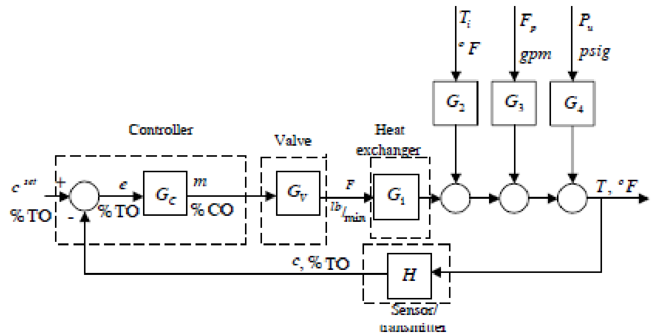


Figure 6-1.7 Block diagram showing control loop and disturbances.

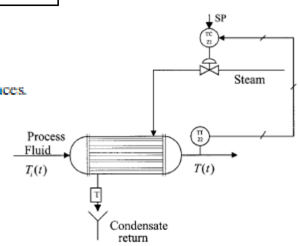


Figure 6-1.1 Heat exchanger control system.

Block Diagrams

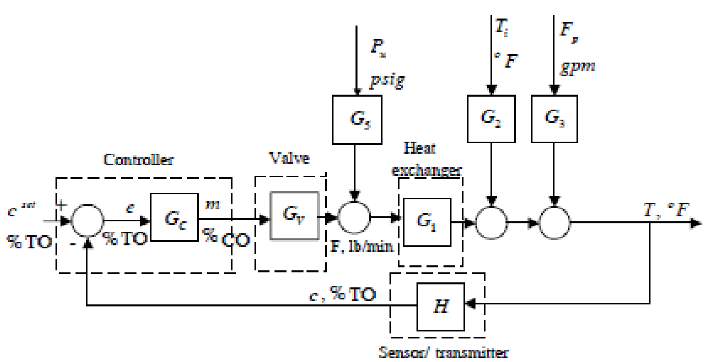


Figure 6-1.8 Block diagram showing control loop and disturbances.

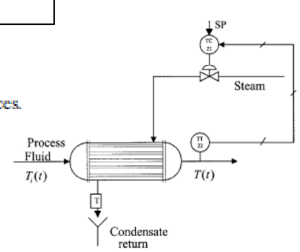


Figure 6-1.1 Heat exchanger control system.

Block Diagrams

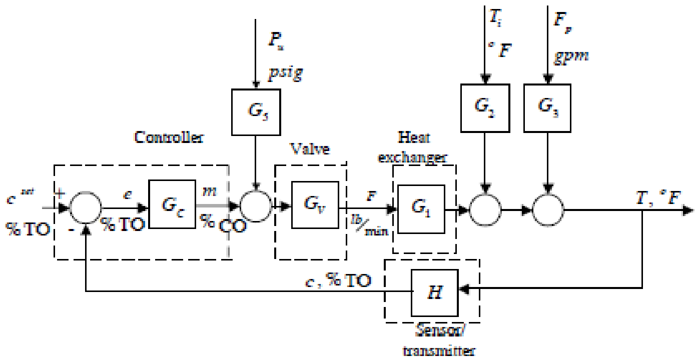


Figure 6-1.9 Another way to draw Fig. 6-1.8.

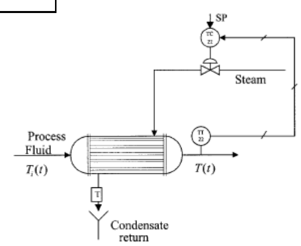


Figure 6-1.1 Heat exchanger control system.

Block Diagrams

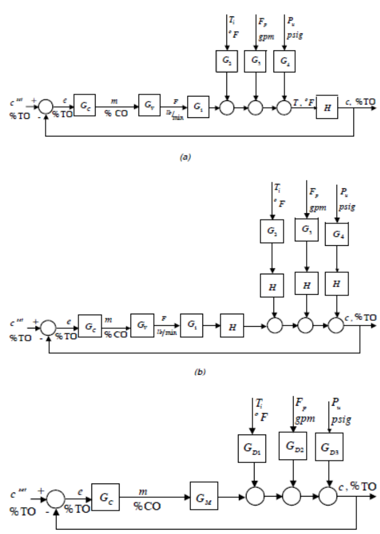


Figure 6-1.10 Algebraic simplification of Fig. 6-1.7.

Block Diagrams

Example 6-1.2. Consider the control system for the drier shown in Fig. 6-1.11, which dries rock pellets. The rock is obtained from the mines, crushed into pellet size, and washed in a water-intensive process. These pellets must be dried before feeding them into a reactor. The moisture of the exiting pellets must be controlled. The moisture is measured and a controller manipulates the speed of the conveyor belt to maintain the moisture at its set point. Let us draw the block diagram for this control scheme.

Block Diagrams

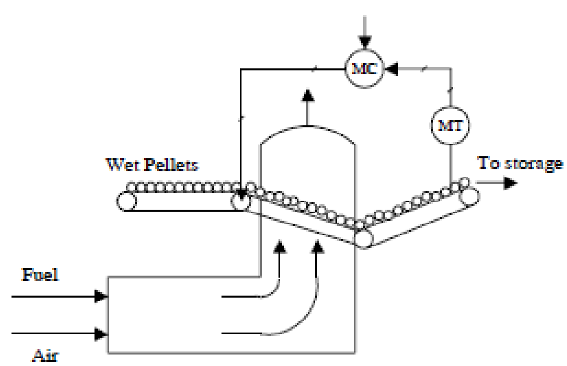
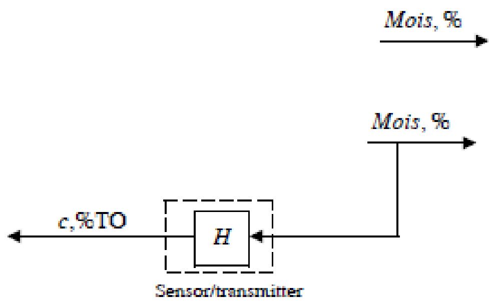


Figure 6-1.11 Phosphate pellets drier.

Block Diagrams



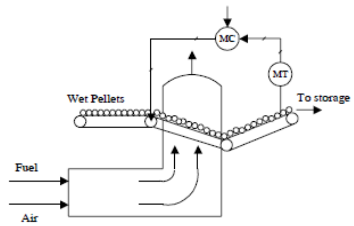
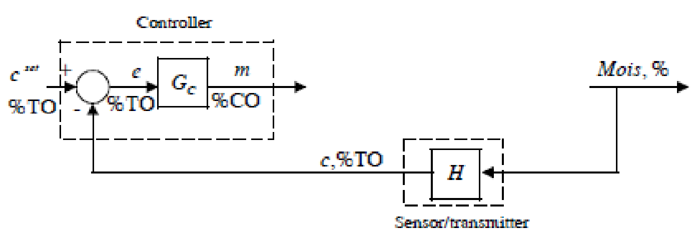


Figure 6-1.11 Phosphate pellets drier.

Block Diagrams



(c)

Figure 6-1.12 Developing the block diagram of the drier control system.

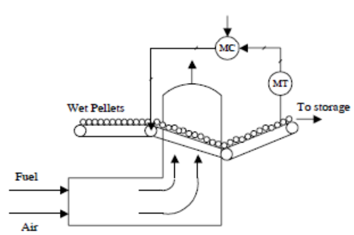
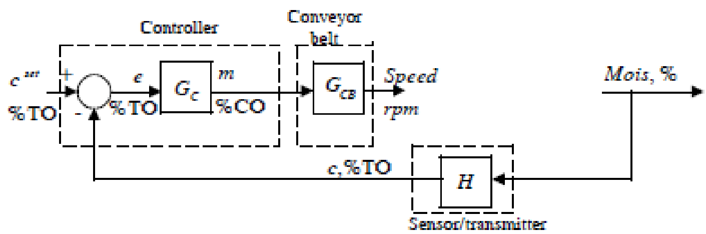


Figure 6-1.11 Phosphate pellets drier.

Block Diagrams



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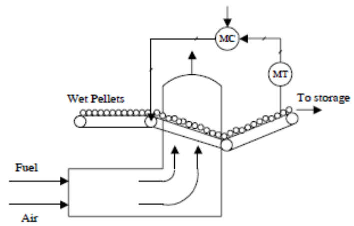
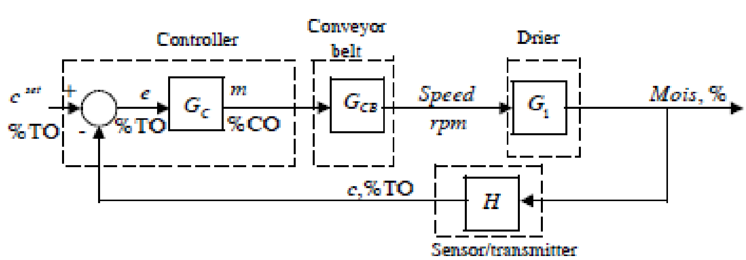


Figure 6-1.11 Phosphate pellets drier.

Block Diagrams



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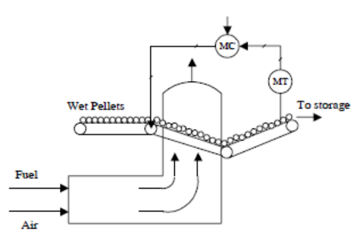

Figure 6-1.11 Phosphate pellets drier.

Block Diagrams

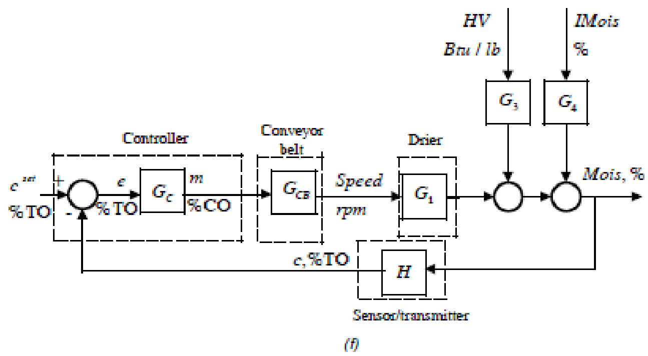

Figure 6-1.12 *Continued.*

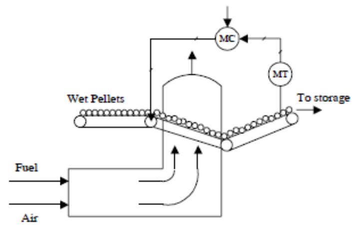

Figure 6-1.11 Phosphate pellets drier.

Control Loop Stability

Once we have learned how to draw block diagrams, the subject of control loop stability can be addressed. We are particularly interested in learning the maximum gain that puts the process to oscillate with constant amplitude. In Chapter 3 we mentioned that this gain is called the ultimate gain, K_{CU}. Above this gain the loop is unstable (you may even say that at this value the loop is already unstable); below this gain the loop is stable.

Control Loop Stability

Let us consider the heat exchanger, shown in Fig. 6-1.1, and its block diagram, shown in Fig. 6-1.7. The transmitter is calibrated from 50 to 150°F. Suppose that the following are the transfer functions of each block in the "loop":

$$G_V = \frac{0.016}{3s+1} \qquad G_1 = \frac{50}{30s+1} \qquad H = \frac{1.0}{10s+1}$$

The time constants are in seconds. The gain of 1.0 in H is obtained by

$$\frac{(100-0)\%\text{TO}}{(150-50)°\text{F}} = \frac{100\%\text{TO}}{100°\text{F}} = 1.0\frac{\%\text{TO}}{°\text{F}}$$

Control Loop Stability

To study the stability of any control system, control theory says that we need only to look at the characteristic equation of the system. For block diagrams such as the one shown in Fig. 6-1.7, the characteristic equation is given by

$$1 + G_C G_V G_1 H = 0$$

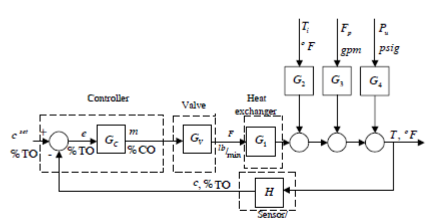

Figure 6-1.7 Block diagram showing control loop and disturbances.

Control Loop Stability

That is, the characteristic equation is given by one (1) plus the multiplication of all the transfer functions in the loop, all of that equal to zero (0). Thus

$$1+\frac{(0.016)(50)(1)G_C}{(3s+1)(30s+1)(10s+1)}=0$$

or

$$900s^3+420s^2+43s+(1+0.8G_C)=0$$

Control Loop Stability

Note that the transfer functions of the disturbances are not part of the characteristic equation, and therefore they do not affect the stability of the loop.

Let us first look at the stability when a P controller is used; for this controller $G_C = K_C$. The characteristic equation is then

$$900s^3+420s^2+43s+(1+0.8G_C)=0$$

Control Loop Stability

This equation is a polynomial of third order; therefore, there are three roots in this polynomial. As we may remember, these roots can be either real, imaginary, or complex. Control theory and mathematics says that for any system to be stable, the real part of all the roots must be negative; Fig. 6-2.1 shows the stability region. Note that the locations of the roots depend on the value of K_C, which is the same thing as saying that the stability of the loop depends on the tuning of the controller.

Control Loop Stability

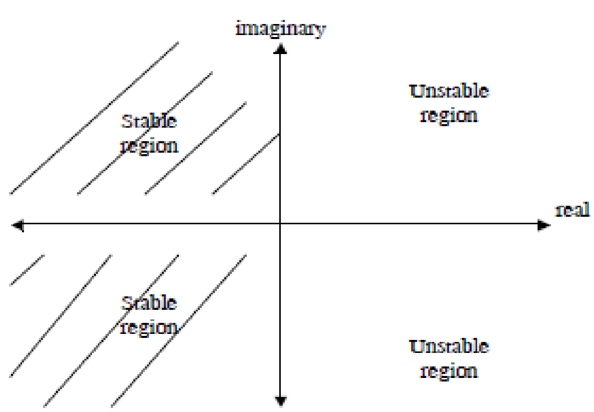

Figure 6-2.1 Roots of the characteristic equation.

Control Loop Stability

If there were two roots on the imaginary axis (they come in pairs of complex conjugates) and all other roots were on the left side of the imaginary axis, the loop would be oscillating with a constant amplitude. The value of K_C that generates this case is K_{CU}.

There are several ways to proceed from Eq. (6-2.4), and control textbooks are delighted to show you so. In this book we are interested only in the final answer, that is, K_{CU}, not in the mathematics.

Control Loop Stability

For this case, which we call the base case, the K_{CU} value and the period at which the loop oscillates, which in Chapter 2 we called the ultimate period T_U are

$$K_{C_U} = 23.8 \frac{\%CO}{\%TO} \quad \text{and} \quad T_U = 28.7\,\text{sec}$$

Let us now learn what happens to these values of K_{CU} and T_U as terms in the loop change.

Effect of Gains

Let us assume that a new transmitter is installed with a range of 75 to 125°F. This means that the transmitter gain becomes

$$\frac{(100-0)\%\text{TO}}{(125-75)^\circ\text{F}} = \frac{100\%\text{TO}}{50^\circ\text{F}} = 2\frac{\%\text{TO}}{^\circ\text{F}}$$

Thus, the transfer function of the transmitter becomes

$$H = \frac{2.0}{10s+1}$$

and the characteristic equation

$$900s^3 + 420s^2 + 43s + (1 + 1.6K_C) = 0$$

Effect of Gains

The new ultimate gain and ultimate period are

$$K_{C_U} = 11.9\frac{\%\text{CO}}{\%\text{TO}} \quad \text{and} \quad T_U = 28.7\,\text{sec}$$

Thus, a change in any gain in the "loop" (in this case we changed the transmitter gain, but any other gain change would have the same effect) will affect K_{CU}. Furthermore, we can generalize by saying that if any gain in the loop is reduced, K_{CU} increases. The reciprocal is also true: If any gain in the loop increases, K_{CU} reduces. The change in gains does not affect the ultimate period.

Effect of Time Constants

Let us now assume that a new faster transmitter (with the same original range of 50 to 150°F) is installed. The time constant of this new transmitter is 5 sec. Thus the transfer function becomes

$$H = \frac{1.0}{5s+1}$$

and the characteristic equation

$$450s^3 + 255s^2 + 38s + (1 + 0.8K_C) = 0$$

Effect of Time Constants

The new ultimate gain and ultimate period are

$$K_{C_U} = 25.7\frac{\%CO}{\%TO} \quad \text{and} \quad T_U = 21.6\,\text{sec}$$

This change in transmitter time constant has affected K_{CU} and T_U. By reducing the transmitter time constant, K_{CU} has increased, thus permitting a higher gain before reaching instability, and T_U has been reduced, thus resulting in a faster loop.

Effect of Time Constants

The effect of a change in any time constant cannot be generalized as we did with a change in gain. Again install the original transmitter, and consider now that a change in design results in a faster exchanger; its new transfer function is

$$G_1 = \frac{50}{20s+1}$$

and the characteristic equation

$$450s^3 + 255s^2 + 38s + (1 + 0.8K_C) = 0$$

Effect of Time Constants

The new ultimate gain and ultimate period are

$$K_{C_U} = 18.7\frac{\%CO}{\%TO} \quad \text{and} \quad T_U = 26.8\text{sec}$$

The effect of a reduction in the exchanger time constant is completely different from that obtained when the transmitter time constant was changed. In this case, when the time constant was reduced, K_{CU} also reduced. It is difficult to generalize; however, we can say that by reducing the smaller (nondominant) time constants, K_{CU} increases, whereas reducing the larger (dominant) time constants, K_{CU} decreases. Usually, the smaller time constants are those of the instrumentation such as transmitters and valves.

Effect of Dead Time

Back again to the original system, but assume now that the transmitter is relocated to another location farther from the exchanger, as shown in Fig. 6-2.2. This location generates a dead time due to transportation. That is, it takes some time to flow from the exchanger to the new transmitter location. Assume that this dead time is only 4sec.

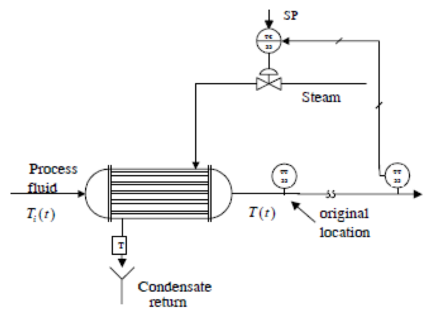

Figure 6-2.2 Heat exchanger showing new transmitter location.

Effect of Dead Time

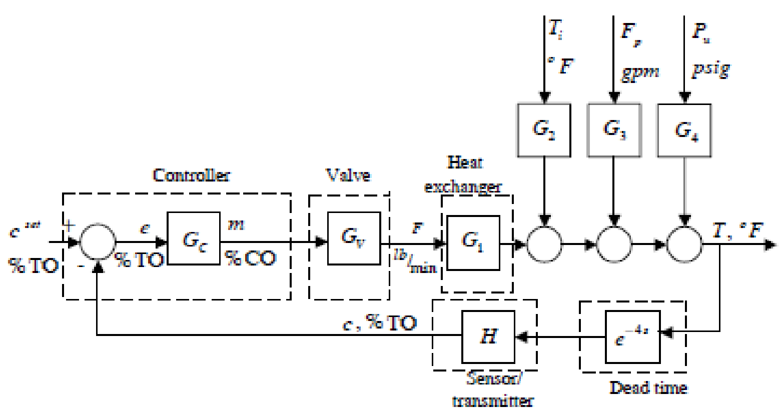

Figure 6-2.3 Block diagram showing dead time.

Effect of Dead Time

The characteristic equation is now

$$900s^3 + 420s^2 + 43s + (1 + 0.8K_C e^{-4s}) = 0$$

The new ultimate gain and ultimate period are

$$K_{C_U} = 9\frac{\%CO}{\%TO} \quad \text{and} \quad T_U = 47.8\,\text{sec}$$

Note the drastic effect of the dead time on K_{CU}. A 4-sec dead time has reduced K_{CU} by 62.2%. T_U was also drastically affected. This proves our comment in Chapter 2 that dead time drastically affects the stability of control loops and therefore the aggressiveness of the controller tunings.

Effect of Integral Action in the Controller

All of the presentation above has been done assuming the controller to be proportional only. A valid question is: How does integration affect K_{CU} and T_U? Even though Ziegler–Nichols defined the meaning of K_{CU} for a P controller only, we will still use it because it still is the maximum gain. The transfer function of a PI controller is given by Eq. (3-2.11):

$$G_C = K_C \frac{\tau_I s + 1}{\tau_I s}$$

Effect of Integral Action in the Controller

and the characteristic equation becomes

$$900s^3 + 420s^2 + 43s + \left(1 + 0.8K_C \frac{\tau_I s + 1}{\tau_I s}\right) = 0$$

Using τ_I = 30sec, the ultimate gain and period are

$$K_{C_U} = 16.2 \frac{\%CO}{\%TO} \quad \text{and} \quad T_U = 34.4\,\text{sec}$$

Thus the addition of integration removes the offset, but it reduces K_{CU}. Integration adds instability to the loop. It also increases T_U, resulting in a slower loop. You may ask yourself: What is the effect of decreasing τ_I? That is, what would happen to K_{CU} if τ_I = 20 sec?

Effect of Derivative Action in the Controller

Now that the effect of integration on the loop stability has been studied, what is the effect of the derivative? Let us look at using a PD controller. The transfer function for a PD controller is given by Eq. (3-2.15):

$$G_C = K_C(\tau_D s + 1)$$

and the characteristic equation becomes

$$900s^3 + 420s^2 + 43s + [1 + 0.8K_C(\tau_D s + 1)] = 0$$

Effect of Derivative Action in the Controller

Using τ_D = 1sec, the ultimate gain and period are

$$K_{CU} = 36.2 \frac{\%CO}{\%TO} \quad \text{and} \quad T_U = 28.2 \text{ sec}$$

Thus the addition of derivative increases the K_{CU} value, adding stability to the loop! From a stability point of view, derivative is desirable because it adds stability and therefore makes it possible to tune a controller more aggressively. However, as discussed in Chapter 3, if noise is present, derivative will amplify it and will be detrimental to the operation.

Effect of Derivative Action in the Controller

In this section we have discussed briefly how to calculate the ultimate gain of a loop. However, we have discussed in more detail how the various gains, time constants, and dead time of a loop affect this ultimate gain. We presented these effects by changing transmitters, process unit (exchanger) design, and so on. *What occurs most commonly, however, is that the process unit itself changes, due to its nonlinear characteristics.*

Summary

In this chapter we have presented the development of block diagrams and discussed the important subject of stability of control loops. These subjects are used in the last chapter of this course.

References

1. Automated Continuous Process Control, Carlos A. Smith, 2002, Wiley-Interscience, ISBN: 978-0471215783.
2. C. A. Smith and A. B. Corripio, Principles and Practice of Automatic Process Control, 3rd ed., Wiley, New York, 2006.